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A PRACTICAL APPROACH TO THE ANALYSIS AND DESIGN OF ZERO VELOCITY ERROR RANGE TRACKERS

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China Lake, California

December 1973

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ABSTRACT

Zero velocity error analog range tracking loops have been in wide use in radar signal processors for many years. However, a general design procedure for calculating the pertinent component values to fulfill a given requirement is apparently nonexistent. This report is intended to fill this void and provide the designer with a practical approach to the design of pertinent component values for second-order, type 2 range tracking integrators. Methods of target thresholding, acquisition, and reacquisition are not discussed.

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Leroy Riggs . . . . Technical Director (Acting)

FOREWORD

Analog range trackers have been in existence for many years; however, no general design procedure for the velocity and range integrators has (as far as the author can determine) ever been presented. This report is intended to supply this information. The theory is put to use in designing a two-bandwidth, second-order, type 2 range tracker. Excellent correlation is obtained between the measured and predicted results.

This work was carried out under Naval Electronics Systems Command Letter PME-198-21; GC; ljt, S-9678 Serial 001512-PME-198-212.

This report was reviewed for technical accuracy by Roger Orr, Mayer Freedman, and Roger Wilcox. The author is indebted to Paul Hilliard for constructing and testing all the track loop circuitry.

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Technical Director (Acting)

CONTENTS

INTRODUCTION

Many radar systems employ range trackers in their video processors. The theory concerning the why's of range tracking is adequately covered in Ref. 1-4. The designer, thus confronted with the need for a range tracker, is apparently at a loss as to how to proceed to design one (a search to find an approach to the design of range trackers has been in vain).

This report presents the basic analysis and design procedures for the design of second-order, type 2 (zero velocity error) range tracking integrators. All necessary design equations are presented, along with a design example.

Acquisition and reacquisition techniques and signal detection criteria are not discussed in detail, as these are subjects encompassing enough to warrant individual treatment. Rather, the discussion assumes that the range tracker is locked on.

BASIC RANGE TRACKING OPERATION

Figure 1 illustrates a simplified pulse radar receiver/video processor. The signal return pulse is processed via a linear automatic gain controlled or logarithmic receiver. The video information is then detected and locked on in range by the target detection and acquisition circuitry. The angle track loop will then be enabled only when an expected return pulse is present.

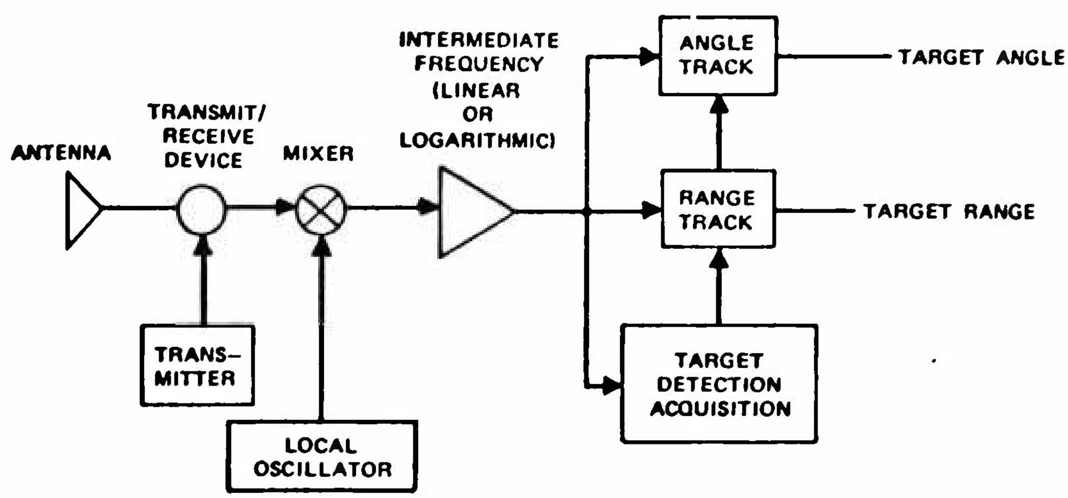


FIG. 1. Basic Pulse Radar Receiver/Video Processor.

The primary function of the range track loop is to track the target return pulses and to enable various functions only during an expected target return (i.e., in certain instances, the receiver may be gated on by the range tracker, thus decreasing the probability of false alarm). The range tracker must be capable of following any movement in range, as would be encountered in aircraft or missile applications. As the airborne radar approaches the target, the range is obviously decreasing; however, the range track loop must still be capable of providing an enable gate to the system. This is the primary reason for utilizing second-order, type 2 (zero velocity error) range trackers. If the radar is traveling at a constant velocity (as is often the case) and if the target should fade, the range enable gates will still be provided until the target reappears or reacquisition is initiated.

Figure 2 illustrates a basic monostatic radar-target configuration.[1] The radar transmits at time T_0, and the transmitted pulse reaches the target at time T_1. The pulse is then reflected and reaches the radar at time T_2. It is obvious that $T_1 = T_2$. Thus, the range to the target, R, (assuming the pulse travels at the speed of light) is

$$R = \frac{C\Delta t}{2} \tag{1}$$

where

C = speed of light (0.984 ft/nsec)
$\Delta t = T_1 + T_2$ (total time of pulse travel)

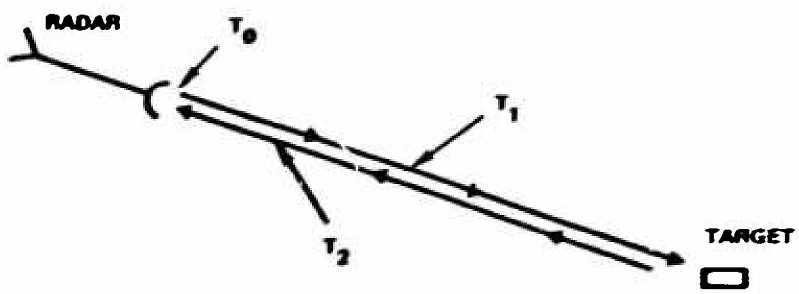


FIG. 2. Typical Radar Target Configuration.

[1] It should be pointed out that all the theory presented will work for bistatic radars as well; however, the time-range equations and implementation will be different from the monostatic case.

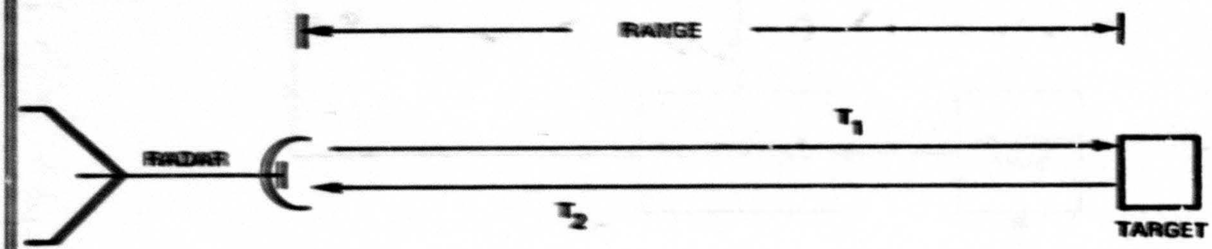
Thus, if a pulse is transmitted at time T_0, and 10 microseconds later the reflected pulse is received, the range to the target is

$$R = \frac{(0.984 \text{ ft/nsec})(10 \times 10^3 \text{ nsec})}{2} \tag{2}$$

or

$$R = 4{,}920 \text{ ft} \tag{3}$$

Figure 3 summarizes the basic time-range equations.



$$\text{Range} = \frac{C\Delta t}{2} \qquad \Delta t = T_1 + T_2$$

$$C = 0.984 \text{ ft/nsec}$$

$$1 \text{ mi} = 5{,}280 \text{ ft} \qquad 1 \text{ nmi} = 6{,}076.1 \text{ ft}$$

$$\Delta t = 10.73\ \mu\text{sec/mi} \qquad \Delta t = 12.35\ \mu\text{sec/nmi}$$

$$\text{Range} = 0.492 \text{ ft/nsec} = 492 \text{ ft/}\mu\text{sec}$$

FIG. 3. Range-Time Relationships (Monostatic Radar).

Figure 4 illustrates the basic components of a range track loop (again, it should be emphasized that target lock-on has occurred). The radar transmits at time T_0 (see Fig. 2). This is termed the master trigger pulse and starts a linear ramp generator. When the ramp generator voltage equals the voltage of the range integrator (range voltage) (Fig. 5a), the comparator is triggered, triggering the range gate which enables the range discriminator. The output of the discriminator (Fig. 5b) is continually "zeroed" by the feedback action of the loop (i.e., the range gate is driven such that the target return is centered in the range gate).

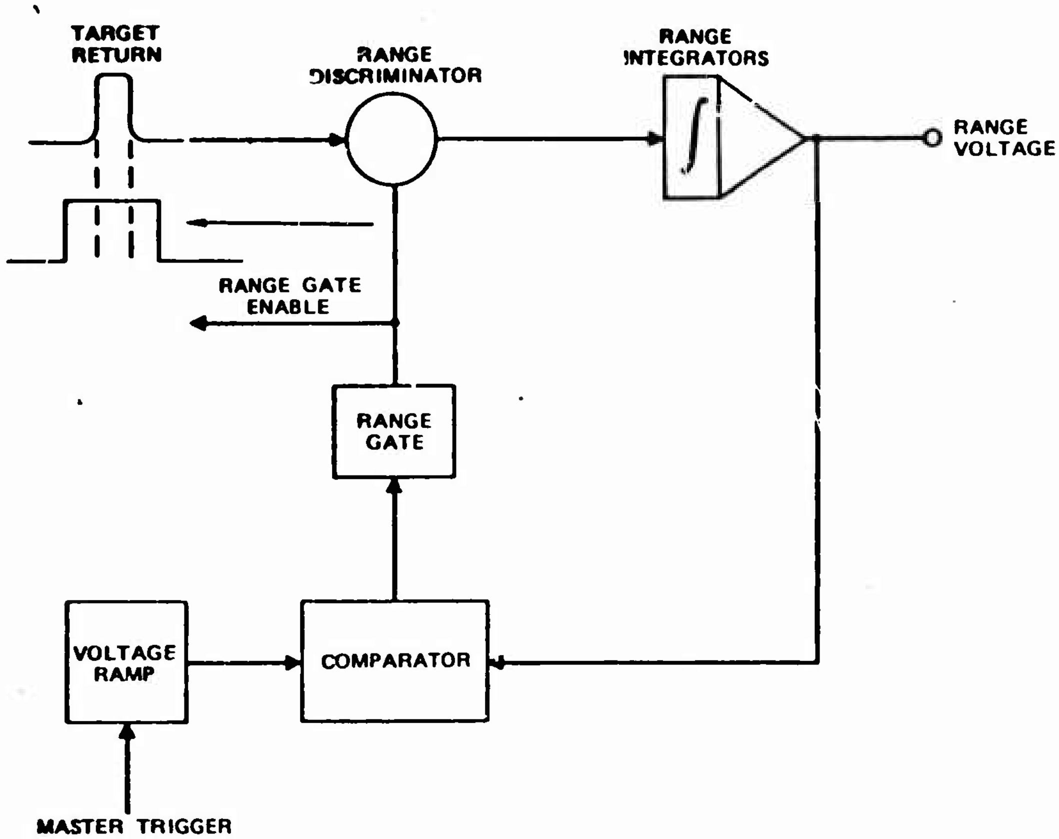
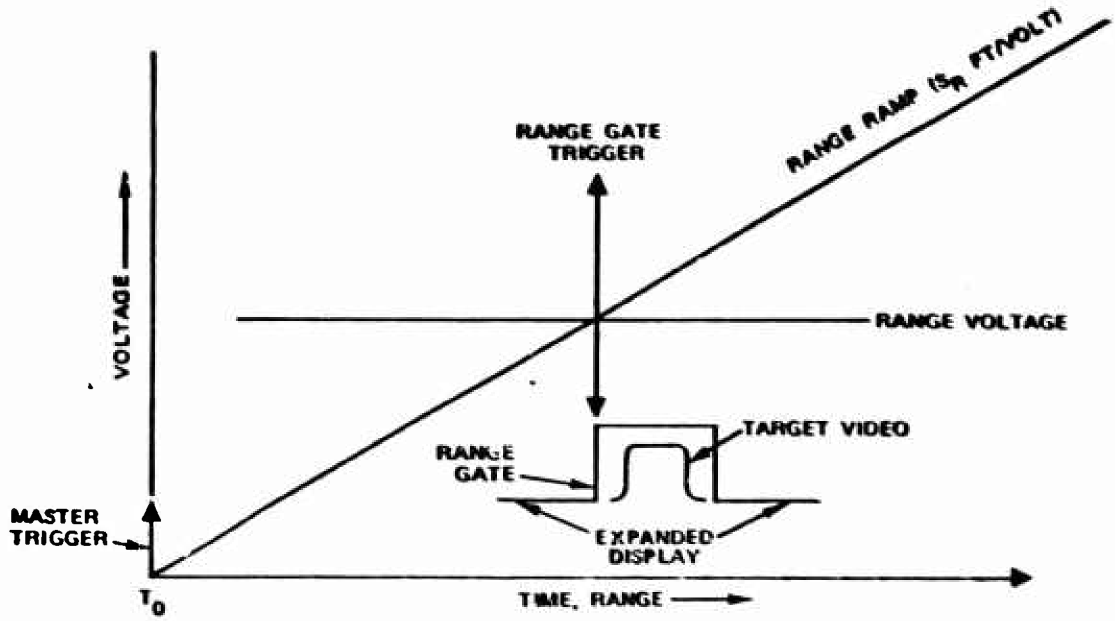


FIG. 4. Basic Range Track Loop.

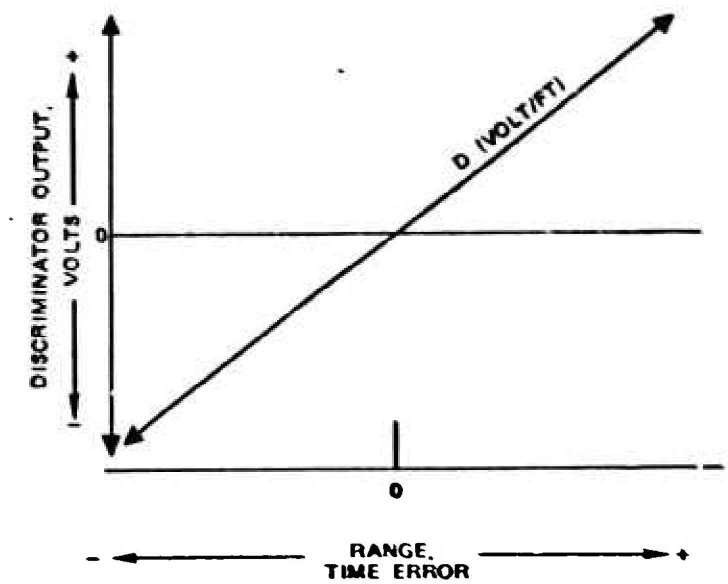
RANGE TRACKER DESIGN ANALYSIS

Figure 6 illustrates the classical second-order, type 2 range tracker. The design of the discriminator, comparator, ramp generator, and early-late gates are critical for a well-designed range track loop. However, these circuits are fairly well established and the design is not discussed here. The main task and objective of this report is to design the range and velocity integrators. Assigning values to the R's and C's is not as straightforward as might appear at first glance.

Figure 7 is a functional diagram of Fig. 6 and is used for analysis. The range error (actual-measured) is amplified by the discriminator, integrated and compared with the actual range. The box marked output scale factor (S_R ft/volt) simply relates range voltage to feet. Figure 7 lends itself to straightforward feedback analysis.



(a) Range ramp-range voltage timing.



(b) Discriminator volt-error curve.

FIG. 5. Range Ramp and Discriminator Timing.

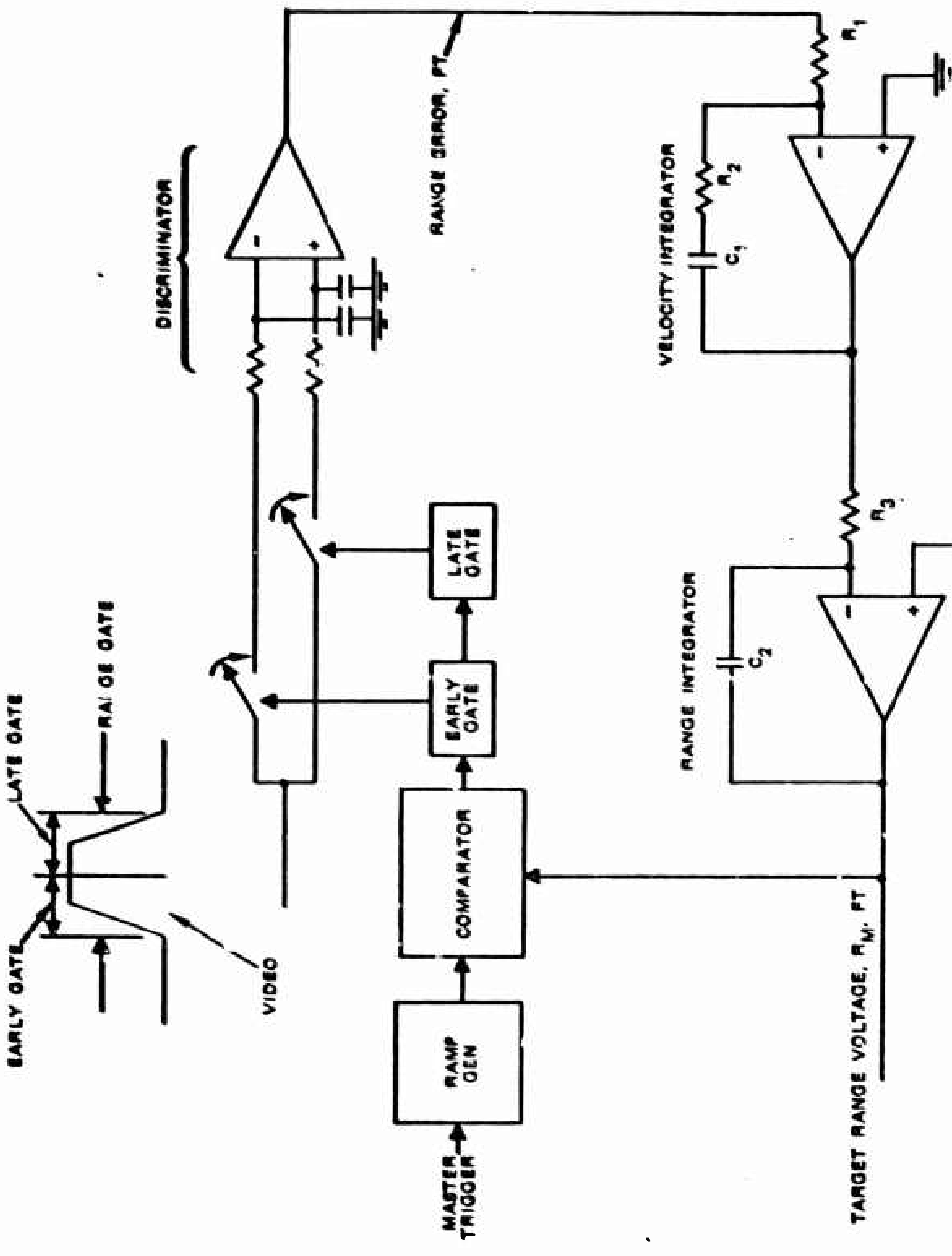


FIG. 6. Seccnd-Order, Type 2 Range Tracker.

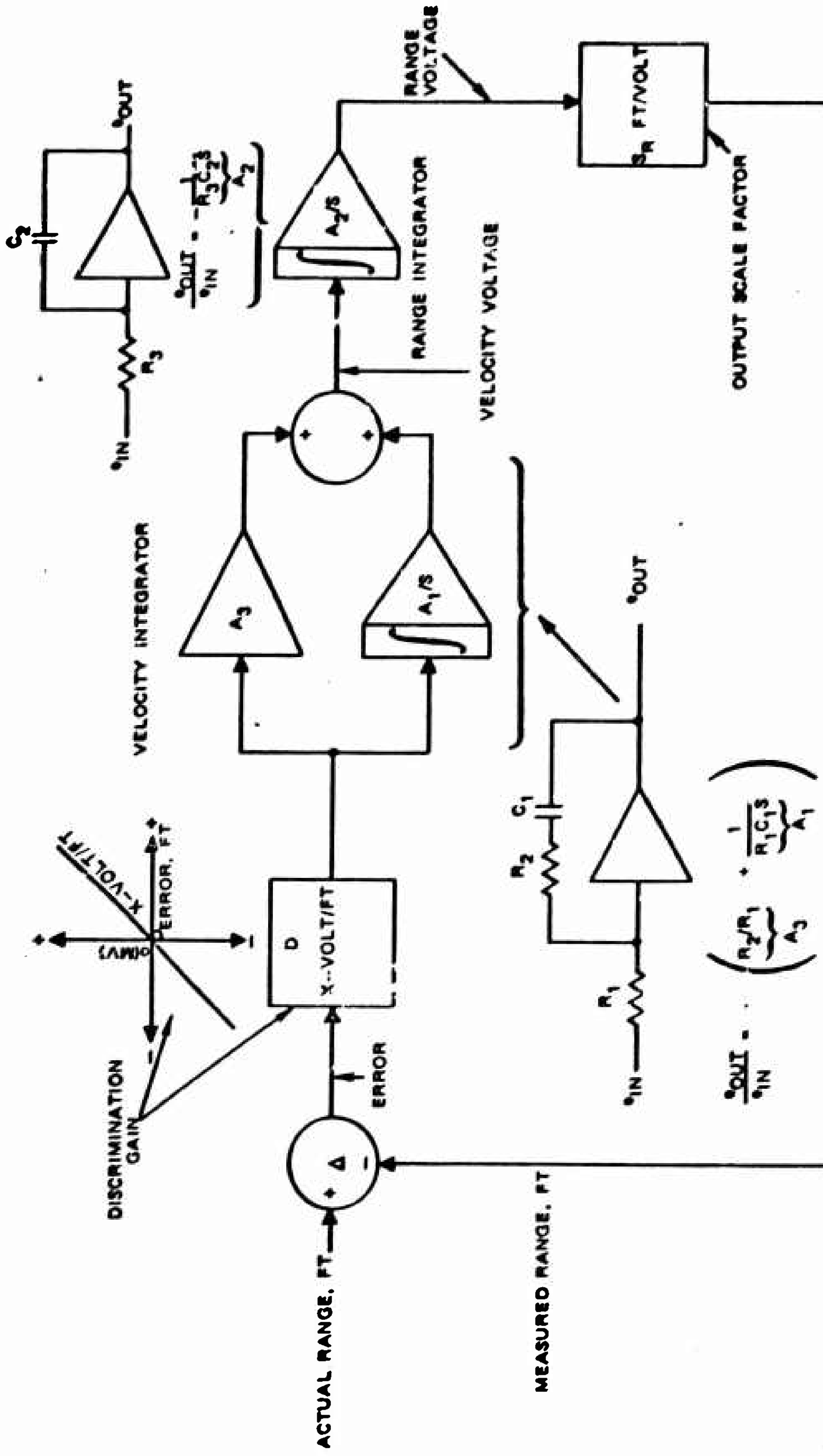


FIG. 7. Functional Second-Order, Type 2 Range Tracker.

The loop gain (LG) for the second-order, type 2 range tracker is

$$LG = D\left(A_3 + \frac{A_1}{S}\right)\left(\frac{A_2}{S}\right) S_R \tag{4}$$

or

$$LG = \frac{A_2A_3DS_R}{S} + \frac{A_1A_2DS_R}{S^2} \tag{5}$$

letting

$$a_1 = A_1A_2S_R \tag{6}$$

$$a_2 = A_2A_3S_R \tag{7}$$

$$LG = \frac{D}{S^2}\left(a_1 + a_2S\right) \tag{8}$$

The discriminator scale factor, D, is left as an independent variable as this term is usually dependent on the instantaneous target video return.

The track loop transfer function is

$$\frac{R_M}{R_A} = \frac{LG}{1 + LG} \tag{9}$$

where

R_M = measured range

and

R_A = actual range

Substituting Eq. 8 into Eq. 9 and simplifying,

$$\frac{R_M}{R_A} = \frac{a_1D + a_2DS}{S^2 + a_2DS + a_1D} \tag{10}$$

The denominator of Eq. 10 is of the classical form of a second-order feedback system

$$S^2 + a_2DS + a_1D = S^2 + 2\delta\omega_n S + \omega_n^2 \tag{11}$$

where

δ = damping ratio
ω_n = loop undamped natural frequency

It is obvious from Eq. 11 that

$$\omega_n = \sqrt{a_1D} \tag{12a}$$

and

$$\delta = \frac{a_2D}{2\omega_n} \tag{12b}$$

The order and type for a feedback system are used indiscriminately by many people. The most common usage is: the order of a system refers to the highest degree of the polynomial expression for the characteristic equation (Eq. 11) and the type of system refers to the number of poles of the LG (Eq. 8) located at the origin. Thus, from Eq. 8 and 11, the range track loop is a second-order, type 2 system. It is well known from classical control theory that a second-order, type 2 system has zero positional and velocity error, and a constant acceleration error.

The similarity of Fig. 7 to that of a classical phase lock loop should be noted. Indeed, if range is substituted for phase, velocity for frequency change, and acceleration for frequency ramp, the basic range tracker loop and phase lock loop are identical. (Appendix A discusses the similarities between the range track loop and the phase lock loop.) The following discussion is based on classical phase lock loop theory.

The design of the range track loop will depend on the maximum range error that can be tolerated for various step position, step velocity, or step acceleration inputs. The error equations may be given as

$$\text{Range error } (\varepsilon) = \frac{\text{Input}(s)}{1 + LG(s)} = \Delta R \tag{13}$$

Substituting Eq. 8 into Eq. 12

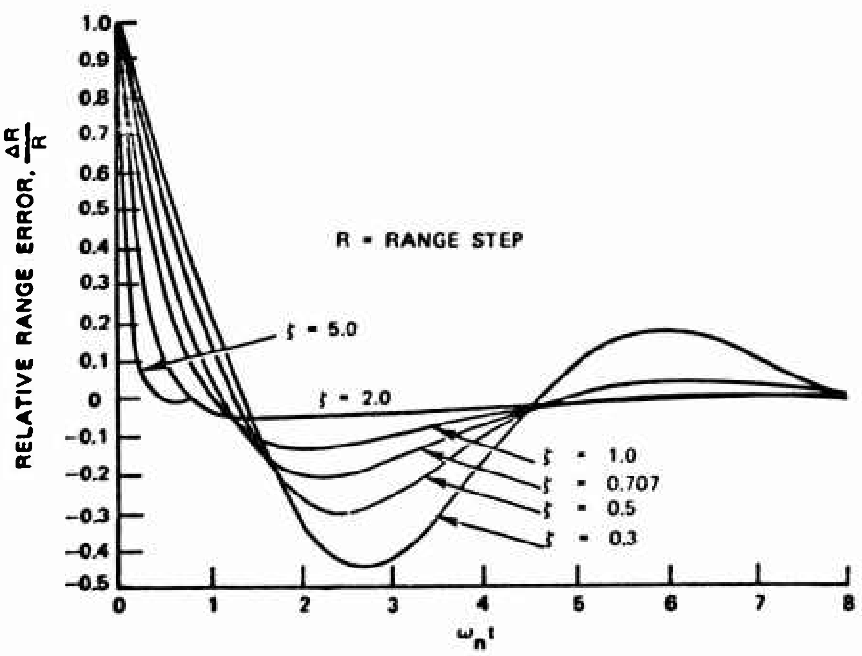
$$\varepsilon(s) = \frac{S^2\,(\mathrm{Input}(s))}{S^2 + a_2 DS + a_1 D} = \Delta R \qquad (14)$$

Substituting the various input functions (step position, velocity, and acceleration) into Eq. 13 and solving for $\varepsilon(s)$ yields very complicated results. However, Ref. 5 has solved and plotted these equations as a function of δ and ω_n, and Fig, 8 illustrates the various normalized error plots.

The effects of the damping factor, δ, on the error response is easily seen from Fig. 8. Classically, the choice of δ depends on the wanted error overshoot, closing time, and bandwidth. The noise bandwidth, BW/N, for a second-order, type 2 system may be given as

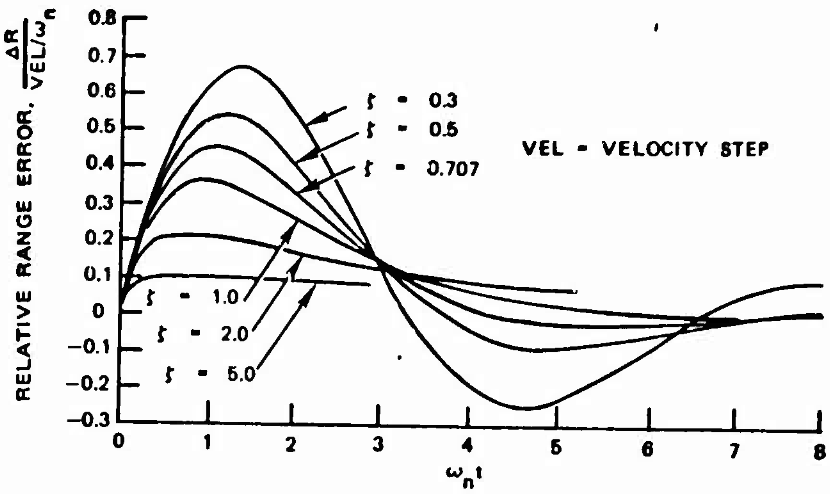
$$BW/N \cong 0.6\omega_n \qquad 0.4 \leq \delta \leq 1 \qquad \text{(Ref. 5)} \qquad (15)$$

and experience has shown that a δ of 0.707 yields near-optimum error response and noise bandwidth.

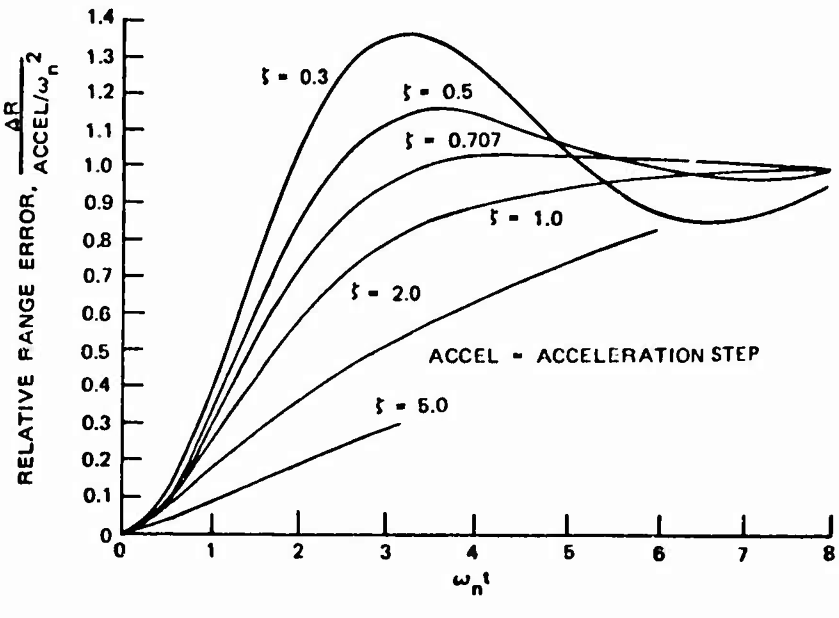


(a) Range.

FIG. 8. Error Response to Step Inputs.



(b) Velocity.



(c) Acceleration.

FIG. 8. (Contd.)

Figure 9 presents the relationships between circuit design parameters (A_1, A_2, A_3, D, and S_R to a_1, a_2, δ and ω_η).

Before discussing the design of the range tracker, the following basic factors must be known:

1. Maximum tracking range
2. Maximum tracker-target velocity
3. Maximum tracker-target velocity step
4. Maximum acceleration due to *g* force
5. Power supply voltages available
6. Maximum range error (ϵ), ΔR.

Many range trackers employ two bandwidths: one wide bandwidth for quick normalization of a velocity step, and after velocity normalization, a slower, narrow bandwidth for normalizing out any expected accelerations. The changing of the bandwidth is easily accomplished by changing R_2 and C_1 (see Fig. 7) via a relay. However, the damping factor, δ, for the two bandwidths should be the same (δ independent of ω_η), and should be kept in mind as we proceed.

RANGE TRACKER DESIGN EQUATIONS

The design of the range integrator component values is straightforward. The maximum tracking range is X-miles (nautical or statute) and the maximum voltage that can be obtained from the range integrator, Y. The range voltage slope (S_v) in volts/second will be

$$\frac{S_v}{\text{nmi}} = \frac{Y}{(12.35 \times 10^{-6})(X\text{-nmi})} \qquad \frac{S_v}{\text{mi}} = \frac{Y}{(10.73 \times 10^{-6})(X\text{-mi})} \tag{16}$$

The range slope, S_R, in foot/volt may now be given as

$$S_R = \frac{4.92 \times 10^8}{S_v} \tag{17}$$

Loop gain (LG) $= \dfrac{A_2A_3DS_R}{S} + \dfrac{A_1A_2DS_R}{S^2}$

Let

$$a_1 = A_1A_2S_R \qquad a_2 = A_2A_3S_R$$

$$LG = \frac{D}{S^2}\left(a_1 + a_2S\right)$$

Range track loop transfer function $\left(\dfrac{R_M}{R_A}\right) = \dfrac{LG}{1 + LG}$

$$\frac{R_M}{R_A}(s) = \frac{a_1D + A_2DS}{S^2 + a_2DS + a_1D}$$

The denominator is of the form

$$S^2 + 2\delta\omega_n S + \omega_n^2$$

or

$$S^2 + a_2DS + a_1D = S^2 + 2\delta\omega_n S + \omega_n^2 = S^2 + \left(A_1A_3S_R\right)DS + \left(A_1A_2S_R\right)D$$

Now

$$\omega_n = \sqrt{a_1D} = \left(\sqrt{A_1A_2S_R}\right)\left(\sqrt{D}\right)$$

$$\delta = \frac{a_1D}{2\omega_n} = \frac{a_2\sqrt{D}}{2\sqrt{a_1}}$$

If a multiple bandwidth system is wanted, δ should be independent of ω_n, therefore

$$\frac{a_2}{\sqrt{a_1}} = \frac{2\delta}{\sqrt{D}} = \text{Constant (K)}$$

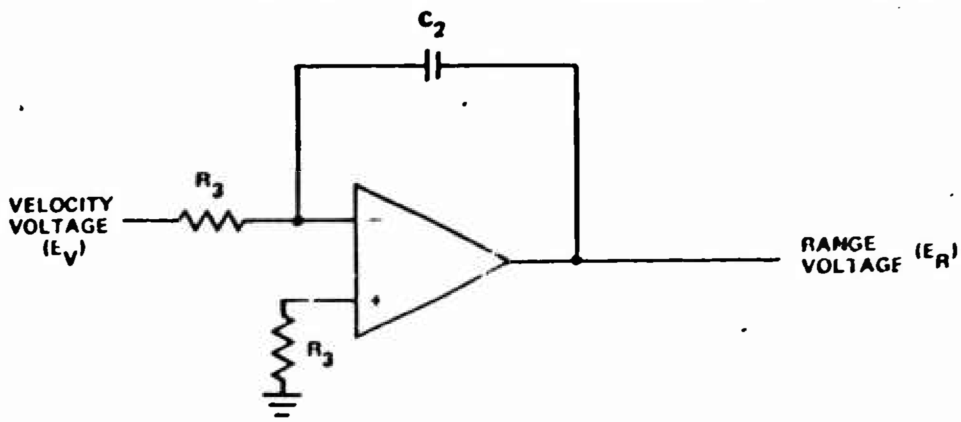
Keeping S_R and A_2 constant (i.e., changing A_3 and A_1)

$$\frac{A_3}{\sqrt{A_1}} = \frac{K}{\sqrt{A_2S_R}}$$

The noise bandwidth, $BW/N \cong 0.6\omega_n$ $\qquad 0.4 \leq \delta \leq 1$

FIG. 9. Second-Order, Type 2 Range Track Loop Analysis Equations.

Since the loop is a second-order, type 2 track loop, the voltage output of the first integrator (or velocity integrator) is linearly related to input velocity. Thus, if the target should fade, the output range voltage will "coast" at the last target velocity. Stated another way, the output will be a linear ramp, the slope of which is dependent on velocity. Figure 10 illustrates the pertinent design equations for the range integrator. A design example will be presented later to demonstrate the design procedures.



The output scale factor is S_R (ft/volt). The maximum output slope occurs for maximum velocity, V_{max}. (The input to the range integrator is E_v/max.)

$$\frac{dE_R/\max}{dt} = \frac{V_{max}}{S_R}\left(\frac{\text{volt}}{\text{sec}}\right)$$

Now

$$\frac{E_v/\max}{R_3} = C_2\left(\frac{V_{max}}{S_R}\right)$$

Therefore

$$R_3C_2 = \frac{\left(E_v/\max\right)\left(S_R\right)}{V_{max}} = \frac{1}{A_2}$$

FIG. 10. Range Integrator Design Equations.

Figure 11 is a plot of the range error, ΔR_v, for normalized velocity step, Vel/ω_η (see Fig. 8b). The necessary ω_η for a given velocity step, Vel, may be found as

$$\omega_\eta/Vel = \frac{X \Delta Vel}{\Delta R_v} \tag{18}$$

where

$$X = \text{value of } \frac{\Delta R_v}{\Delta Vel/\omega_\eta} \text{ from Fig. 11 for a given } \delta$$

ΔR_v = maximum permissible range error for velocity step

ΔVel = maximum velocity step

The design equations for R_1, R_v, and C_v can now be found by solving Eq. 6, 7, and 11. The results are illustrated in Fig. 12.

Figure 13 illustrates the range error, ΔR_a, for normalized acceleration step (see Fig. 8c). The natural frequency for a practical acceleration input is usually smaller than for a velocity step. Therefore, the design for a given acceleration is termed the narrow-band mode. The loop natural frequency may be found, using Fig. 12, once the maximum range error, acceleration, and δ are known.

$$\omega_\eta/Accel = \sqrt{\frac{X\ Accel}{\Delta R_a}} \tag{19}$$

Now, a_1 may be found similarly as in the wideband mode

$$\frac{a_1}{Accel} = \frac{\omega_\eta^2/Accel}{D} \tag{20}$$

or

$$a_1/Accel = \frac{X\ Accel}{\Delta R_a D} \tag{21}$$

where

$$X = \text{value of } \frac{\Delta R_a}{\text{Accel}/\omega_n} \text{ from Fig. 13 for a given } \delta$$

ΔR_a = maximum permissible range error for acceleration step

Accel = Acceleration step

It was mentioned that the damping, δ, should be independent of ω_n. Substituting Eq. 12a into Eq. 12b and solving for δ,

$$\delta = \frac{a_2\sqrt{D}}{2\sqrt{a_1}} \tag{22}$$

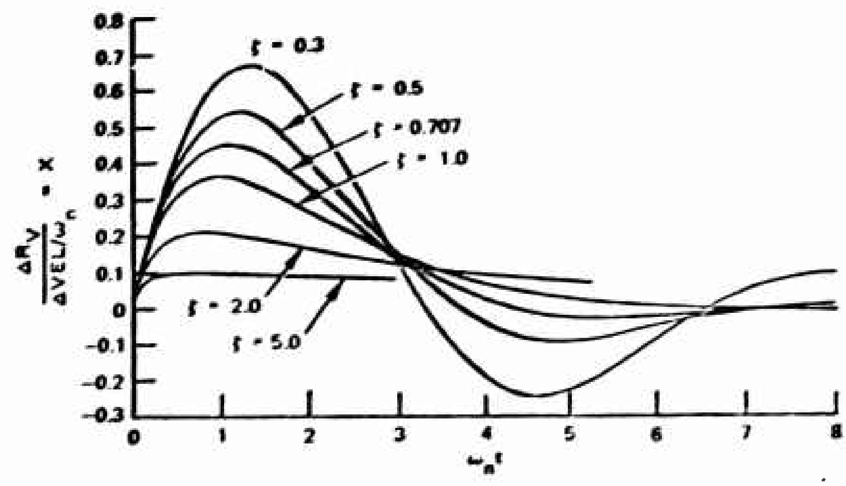
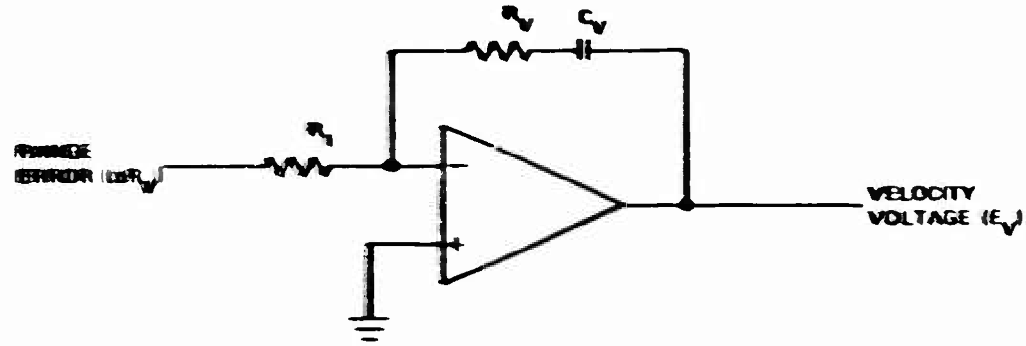
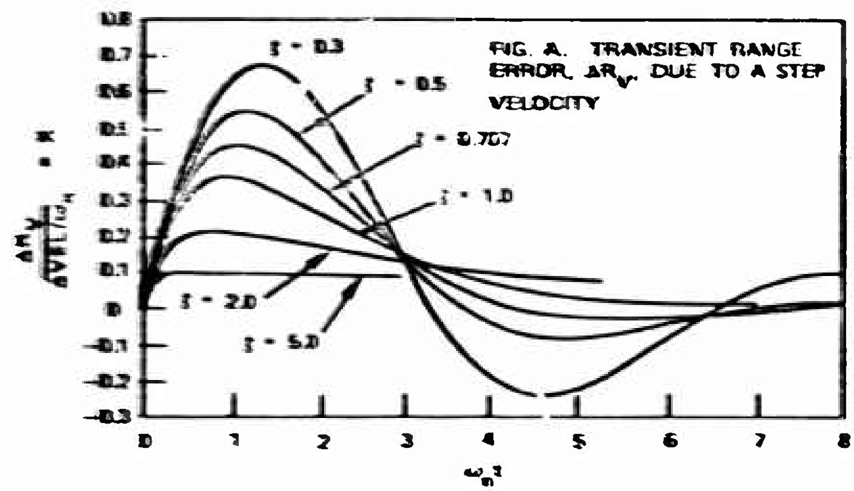


FIG. 11. Transient Range Error, ΔR_v, Due to a Normalized Step Velocity.

Since D is independent of bandwidth,

$$\frac{a_2/\text{Vel}}{\sqrt{a_1/\text{Vel}}} = \frac{2\delta}{\sqrt{D}} = \text{Constant (K)} \tag{23}$$





For a given step input velocity, find (for a given δ and range error ΔR_T) ω_η/Vel.

$$\omega_\eta/\mathrm{Vel} = \frac{K\Delta \mathrm{Vel}}{\Delta R}$$

where K is read from Fig. A.

$$a_1/\mathrm{Vel} = \frac{\omega_\eta^2/\mathrm{Vel}}{D} \qquad a_2/\mathrm{Vel} = \frac{2\delta\omega_\eta/\mathrm{Vel}}{D}$$

$$R_1 C_V = \frac{1}{A_1/\mathrm{Vel}} = \frac{A_2 D S_R}{\omega_\eta^2/\mathrm{Vel}} \qquad \frac{R_V}{R_1} = A_3/\mathrm{Vel} = \frac{a_2/\mathrm{Vel}}{A_2 S_R}$$

The closure time may be found, knowing δ and ω_η, from Fig. A.

FIG. 12. Velocity Integrator Design Equations (Wideband).

Thus, once a_2 and a_1 have been calculated for the wideband mode (velocity step), the constant, K, has been determined. To solve for the narrow-band a_2, simply solve Eq. 23

$$a_2/\text{Accel} = \sqrt{Ka_1/\text{Accel}} \tag{24}$$

Figure 14 summarizes the design equations for the narrow-band (acceleration) mode.

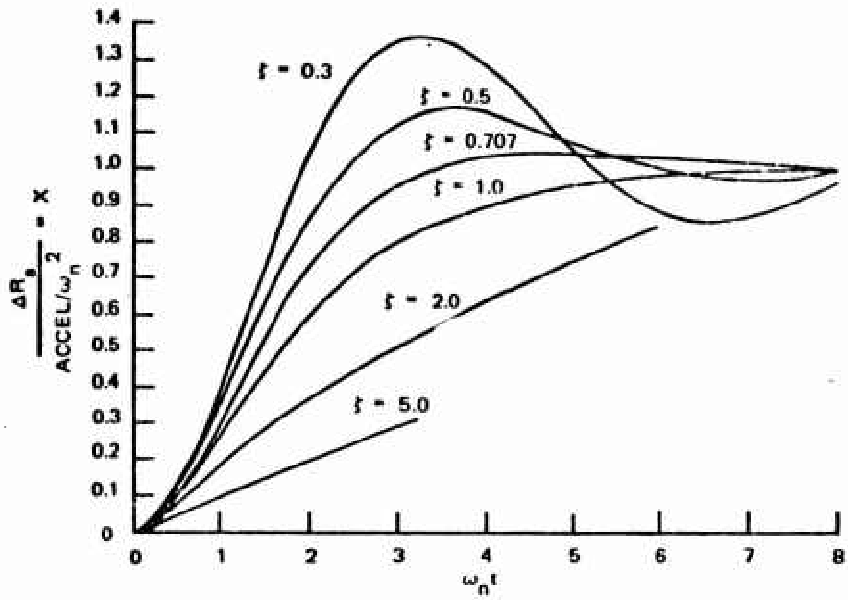
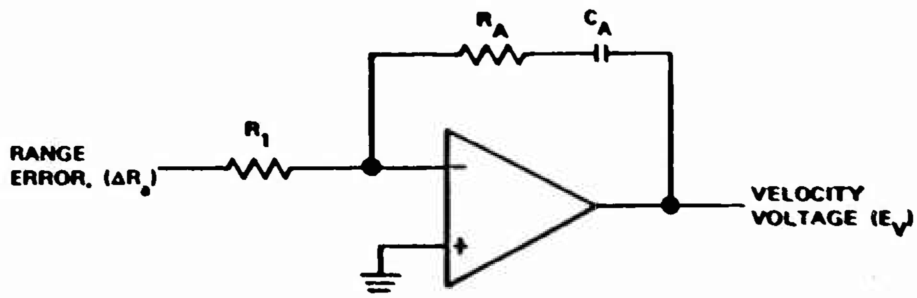


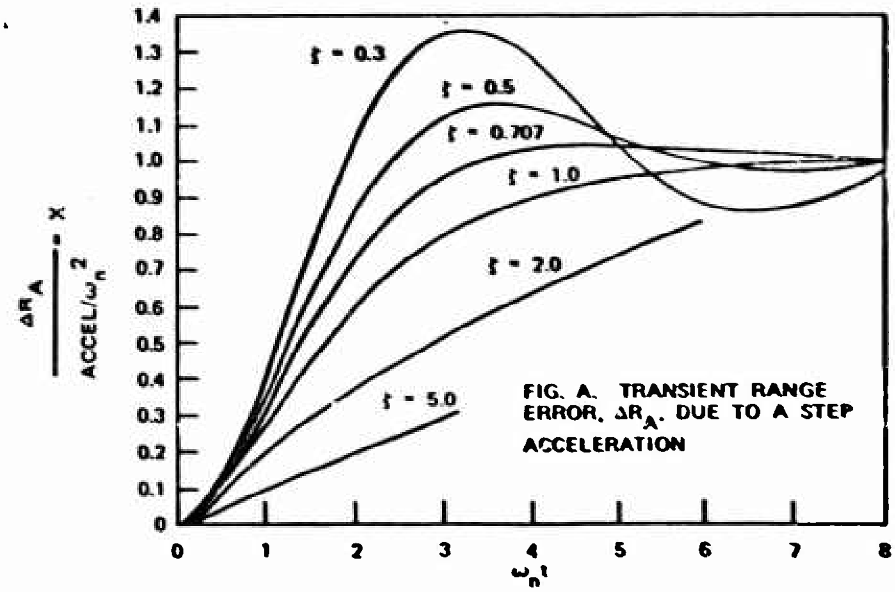
FIG. 13. Transient Range Error, ΔR_a, Due to a Normalized Acceleration Step.

DESIGN EXAMPLE

Now that the design equations have been presented, a typical range tracker will be designed to illustrate the usefulness and validity of these equations. The range tracker will be designed according to the following specifications:

Maximum closing velocity, ft/sec	1,250
Maximum acceleration, *g* (180 ft/sec^2)	5.6
Maximum range, nmi	30
Maximum velocity step ΔVel, ft/sec	400
Maximum voltage for range ramp, volts	15
Range gate width, nsec	100
Maximum range error, ft	$\Delta R_V = 10$; $\Delta R_A = 5$





For a given step acceleration, find (for a given δ and range error ΔR_A) ω_η/Vel,

$$\omega_\eta/\mathrm{Accel} = \sqrt{\frac{X\ \mathrm{Accel}}{\Delta R_A}}$$

where X is read from Fig. A.

$$a_1/\mathrm{Accel} = \frac{X\ \mathrm{Accel}}{(\Delta R_A)D} \qquad a_2/\mathrm{Accel} = K\sqrt{a_1/\mathrm{Accel}}$$

where

$$K = \frac{a_2/\mathrm{Vel}}{\sqrt{a_1/\mathrm{Vel}}} = \frac{2\delta}{\sqrt{D}} \qquad R_1C_A = \frac{A_2S_R}{a_1/\mathrm{Accel}} = \frac{1}{A_1} \qquad \frac{R_A}{R_1} = \frac{a_2/\mathrm{Accel}}{A_2S_R} = A_3$$

The closure time may be found, knowing δ and ω_η, from Fig. A.

Fig. 14. Velocity Integrator Design Equation (Narrow Band).

DESIGN PROCEDURE

Determine Linear Ramp Width (see Fig. 3)

$$1 \text{ nmi} = 12.35\ \mu\text{sec}$$

therefore

$$\Delta t = \left(12.35 \frac{\mu\text{sec}}{\text{nmi}}\right)(30 \text{ nmi}) = 370.5\ \mu\text{sec} \tag{25}$$

Determine the Range Slope, S_R (Eq. 16 and 17)

Since

$$S_v = \frac{Y \text{ (volts)}}{(12.35 \times 10^{-6})(X\text{-nmi})} \tag{26}$$

and

$$S_R = \frac{4.92 \times 10^8}{S_v} \tag{27}$$

$$S_R = \frac{(6.076 \times 10^3)(X\text{-nmi})}{Y \text{ (volts)}} \tag{28}$$

$$S_R = \frac{(6.076 \times 10^3)(30)}{15} = 12.15 \times 10^3 \text{ ft/volt} \tag{29}$$

or

$$S_R = 12.15 \text{ ft/millivolt} \tag{30}$$

Design Range Integrator (see Fig. 10)

Once the range slope, S_R, is known it is a simple matter to define the range integrator.

$$R_3C_2 = \frac{(E_v/\text{max})\ (S_R)}{\text{Vel/max}} \tag{31}$$

where

E_v/max = velocity integrator output for Vel/max

Vel/max = maximum expected velocity

The value for E_v/max is open for choice. This voltage may be used for other system functions, so a large voltage is usually specified. Let

$E_v/max = 10$ volts

Now

$$R_3C_2 = \frac{(12.15 \times 10^3)(10)}{1250} \quad (32)$$

$$R_3C_2 = 97.2 \quad (33)$$

or

$$A_2 = \frac{1}{R_3C_2} = 0.0103 \quad (34)$$

It is usually desirable to solve for R_3 after choosing a C_2, since capacitor values are restricted.

It is obvious that A_2 is too small, since a C_2 of 1 microfarad yields $R_3 = 97$ megohms. It is a simple task, however, to scale R_3C_2 by placing an attenuator in front of the integrator (Fig. 15). Resistors R_a and R_b attenuate the input voltage to the integrator to yield more reasonable values for R_3 and C_2. Let

$$R_3 = 1\ M\Omega \qquad C_2 = 2.2\ \mu F \quad (35)$$

Now, from Eq. 30, substituting E_v' for E_v,

$$E_v' = \frac{(R_3C_2)\ Vel/max}{S_R} \quad (36)$$

or

$$E_v' = \frac{(2.2)(1250)}{12.15 \times 10^3} = 0.227 \tag{37}$$

Thus, the attenuation needed is

$$\text{Atten} = \frac{E_v'}{E_v} = \frac{0.227}{10} = 0.0227 \tag{38}$$

Let

$$R_s = 1K\ \Omega$$

$$R_b = \frac{R_s(1-\text{Atten})}{\text{Atten}} = 43.2K\ \Omega \tag{39}$$

$$A_2 = \text{Atten}\left(\frac{1}{R_3C_2}\right) = 0.0227\left(\frac{1}{2.2}\right) = 0.0103 \tag{40}$$

which compares with Eq. 34.

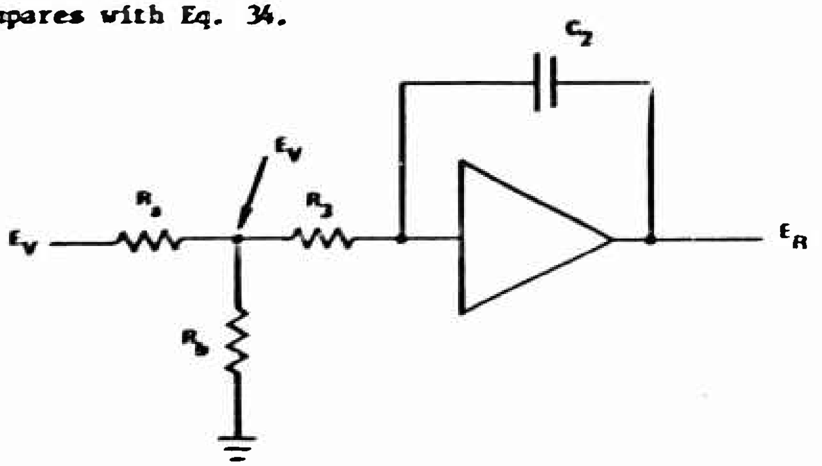


FIG. 15. Scaled Range Integrator.

<u>Design Wideband Velocity Integrator (see Fig. 11)</u>

A value of $\delta = 0.7$ will be chosen as this yields near-optimum noise bandwidth and transient response. The maximum range error, ΔR_v, will be 10 feet. Using Fig. 12,

$$\omega_n/\text{Vel} = \frac{X\Delta\text{Vel}}{\Delta R_v} \tag{41}$$

where

X = maximum normalized error for a given δ.

Thus, for $\delta = 0.7$

$$X = 0.46 \tag{42}$$

and

$$\omega_\eta/Vel = \frac{(0.46)400}{10} = 18.4 \text{ rad/sec} \tag{43}$$

The noise bandwidth is

$$BW/N/Vel = 0.6\omega_\eta/Vel = 11.04 \text{ Hz} \tag{44}$$

$$a_1/Vel = \frac{\omega_\eta^2/Vel}{D} = \frac{338.6}{D} \tag{45}$$

$$a_2/Vel = \frac{2\delta\omega_\eta/Vel}{D} = \frac{26}{D} \tag{46}$$

Experience has shown that a discriminator gain, D, of 50 millivolts/foot will yield realizable values for R_1, R_v, and C_v.

$$R_1C_v = \frac{A_2DS_R}{\omega_\eta^2/Vel} = \frac{(0.0103)(0.05)(12.15 \times 10^3)}{338} \tag{47}$$

or

$$R_1C_v = 0.0185 = \frac{1}{A_1/Vel} \tag{48}$$

$$\frac{R_v}{R_1} = \frac{a_2/Vel}{A_2S_R} = \frac{520}{(0.0103)(12.15 \times 10^{-3})} \tag{49}$$

or

$$\frac{R_V}{R_1} = 4.163 = A_3/Vel \qquad (50)$$

Values for R_1, R_V, and C_V will be solved for after the narrow-band (acceleration) values have been found.

Design Narrow-Band Acceleration Integrator (see Fig. 14)

$$\omega_\eta/Accel = \sqrt{\frac{X\ Accel}{\Delta R_A}} \qquad (51)$$

where

$X = 1.05$ for $\delta = 0.707$

$\Delta R_A = 5$ ft (Note that this is smaller than ΔR_V, which is 10 feet.)

$$\omega_\eta/Accel = \sqrt{\frac{(1.05)(180)}{5}} = 6.15 \text{ rad/sec} \qquad (52)$$

$$BW/N/Accel \cong 0.6\omega_\eta/Accel = 3.68 \text{ Hz} \qquad (53)$$

$$a_1/Accel = \frac{X\ Accel}{\Delta R_A D} = \frac{(1.05)(180)}{(5)(0.05)} = 756 \qquad (54)$$

$$K = \frac{2\delta}{\sqrt{D}} = \frac{1.4}{0.223} = 6.26 \qquad (55)$$

$$a_2/Accel = K\sqrt{a_1/Accel} = 6.26\sqrt{756} = 172 \qquad (56)$$

Now

$$R_1C_A = \frac{A_2S_R}{a_1/Accel} = \frac{(0.0103)(12.15 \times 10^3)}{756} = 0.166 \qquad (57)$$

$$\frac{R_A}{R_1} = \frac{a_2/\text{Accel}}{A_2 S_R} = \frac{172}{(0.0103)(12.15 \times 10^3)} = 1.37 \qquad (58)$$

Let

$$C_A = 2.2\ \mu F$$

$$R_1 = \frac{0.166}{2.2 \times 10^{-6}} = 75.5K\ \Omega \qquad (59)$$

Now, from Eq. 58

$$R_A = (1.37)(75.5K\ \Omega) = 103.4K\ \Omega \qquad (60)$$

Now, knowing R_1, R_v, and C_v, may be found from Eq. 50

$$R_v = (4.16)(75.5K\ \Omega) = 314.3K\ \Omega \qquad (61)$$

From Eq. 48

$$C_v = \frac{0.0185}{75.5\ K} = 0.25\ \mu F \qquad (62)$$

Figure 16 illustrates the second-order, type 2 analog range tracking integrators.

Table 1 summarizes the design calculations.

The discriminator gain, D, is seldom constant in most designs (D will usually be at least somewhat dependent on the received signal input intensity). Figures 17a and 17b illustrate the effect of varying D on the natural frequency, ω_n, and damping, δ.

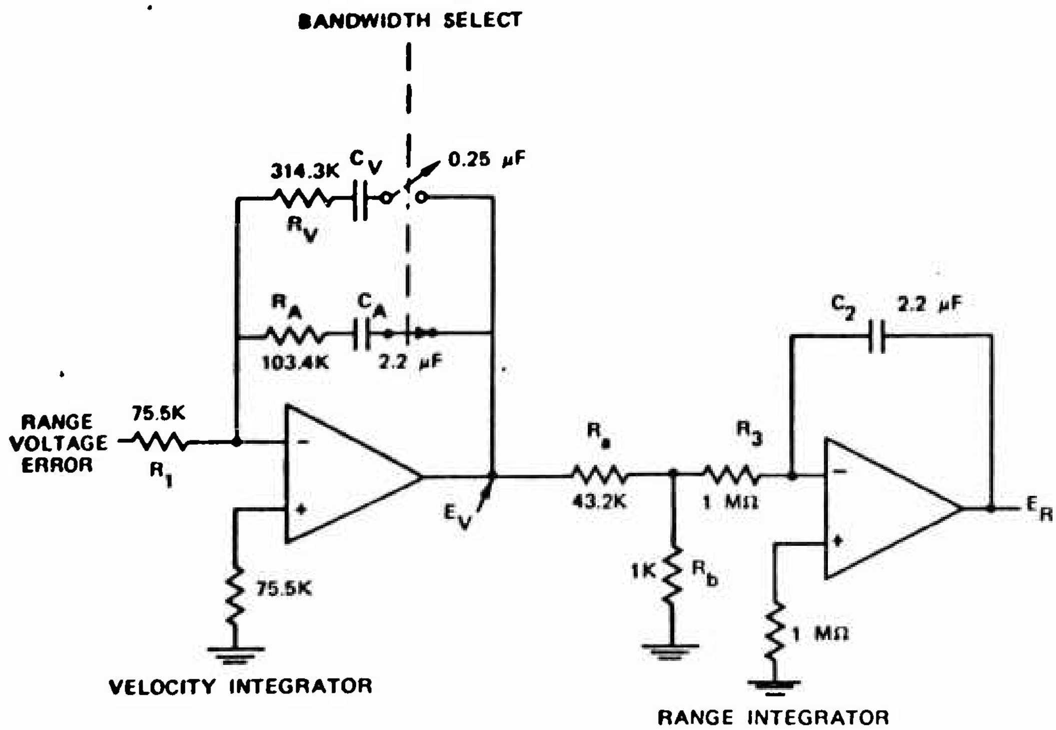
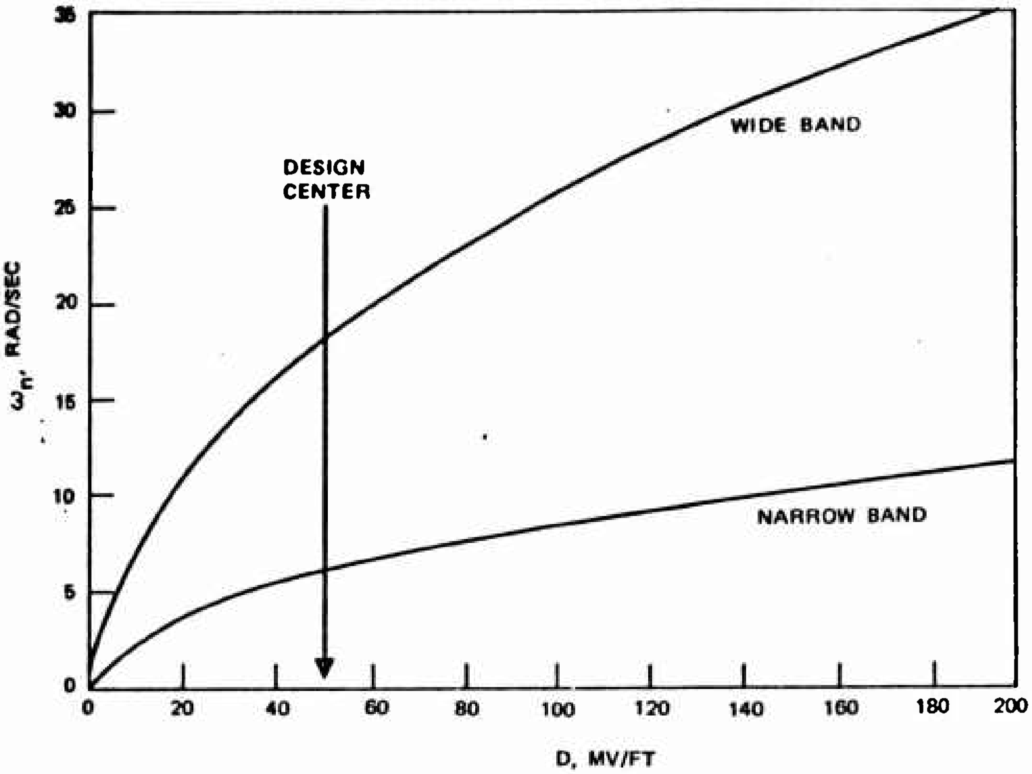


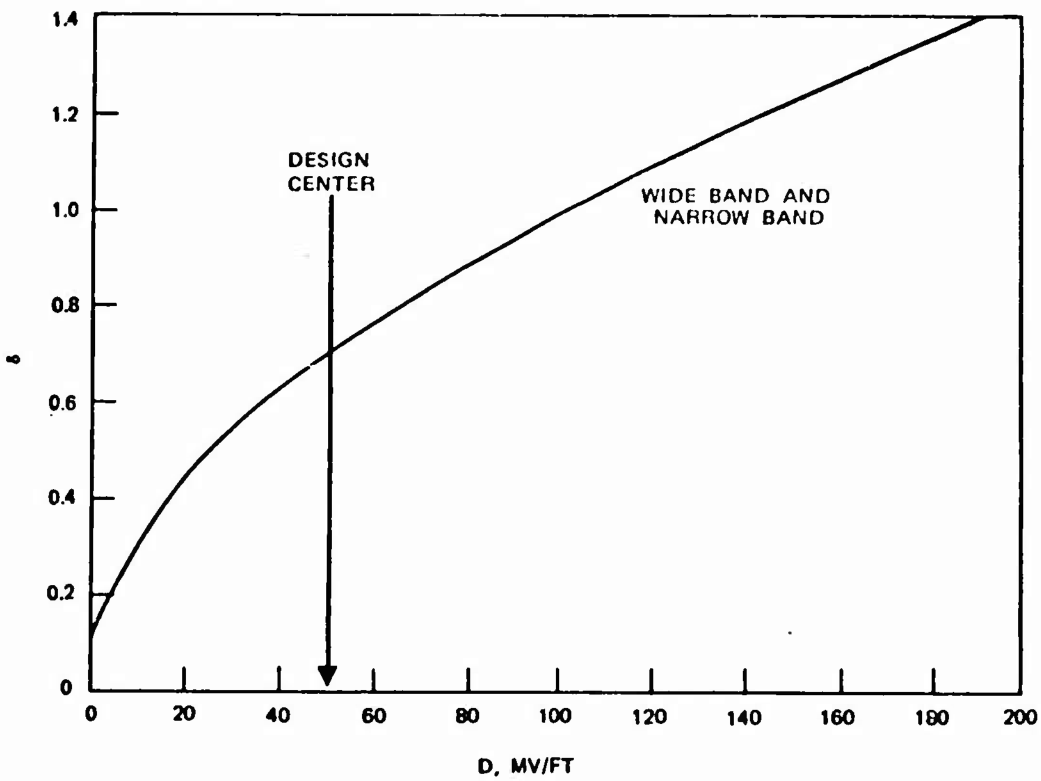
FIG. 16. Second-Order, Type 2 Range Tracking Integrator.

TABLE 1. Design Summary.

Wide bandwidth	Narrow bandwidth
$A_2 = .0103$	$A_2 = 0.0103$
$\omega_n = 18.4$ rad/sec	$\omega_n = 6.15$ rad/sec
BW/N = 11.04 Hz	BW/N = 3.68 Hz
$A_1 = \frac{1}{R_1 C_v} = 54.05$	$A_1 = \frac{1}{R_1 C_a} = 6.02$
$A_3 = \frac{R_v}{R_1} = 4.16$	$A_3 = \frac{R_a}{R_1} = 1.37$



(a) ω_η versus D.



(b) δ versus D.

FIG. 17. Effect of Varying D on Natural Frequency, ω_η, and Damping, δ.

DESIGN RESULTS

Figure 18 illustrates the experimental range track loop (Appendix B contains the schematics for the early/late gates, discriminator, ramp generator, and comparator). The measured output range slope, S_R, is 12,800 foot/volt, which compares favorably with the predicted value (Eq. 29) of 12,150 foot/volt. (This deviation can easily be adjusted if one wishes to place a potentiometer in the ramp generator.)

Figure 19 illustrates the velocity integrator output voltage versus velocity. Again, the small difference between measured and predicted is due to the deviation in S_R.

The track loop was designed for a discriminator gain, D, of 50 millivolts/foot; however, the measured discriminator gain is 35 millivolts/foot. This deviation may also be adjusted if a potentiometer is placed in the discriminator. However, this deviation only slightly degrades ω_η and δ (see Fig. 17). The predicted values for ω_η and δ for a discriminator gain of 35 millivolts/foot are:

Wide band	$\omega_\eta = 15$ rad/sec	$\delta = 0.6$
Narrow band	$\omega_\eta = 5$ rad/sec	$\delta = 0.6$

Figure 20 shows the error voltage as a function of time for a 300-foot/second velocity step. A comparison of this figure and the theoretical range error (see Fig. 11) shows the system is indeed slightly underdamped ($\delta \cong 0.6$). The measured values for ω_η and δ are:

Wide band	$\omega_\eta \cong 15$ rad/sec	$\delta \cong 0.6$
Narrow band	$\omega_\eta \cong 5.5$ rad/sec	$\delta \cong 0.6$

The important point is the measured and predicted values for ω_η and δ are very close (the value for D was increased to 50 millivolts/foot, and indeed the values for ω_η and δ approached the designed values shown in Fig. 17).

Figure 21 illustrates the effect of decreasing the video input pulse (thus D). D was decreased to 15 millivolts/foot. The decrease in δ is obvious ($\delta \cong 0.4$). Again, δ is independent of bandwidth and ω_η. The predicted (see Fig. 17) and measured[2] values for ω_η are:

	ω_η		δ	
	Measured	Predicted	Measured	Predicted
Wide band	10.7 rad sec	10 rad/sec	4	3.8
Narrow band	2.8 rad/sec	3 rad/sec	4	3.8

[2]The measured value for ω_η is easily found by noting the time of the first zero crossing of the error curve. This gives the value $\omega_\eta t$. Thus, simply divide the predicted value for $\omega_\eta t$ (see Fig. 8b) by the measured time to find ω_η. The value for δ can be closely measured by comparing the undershoot with that of Fig. 8b.

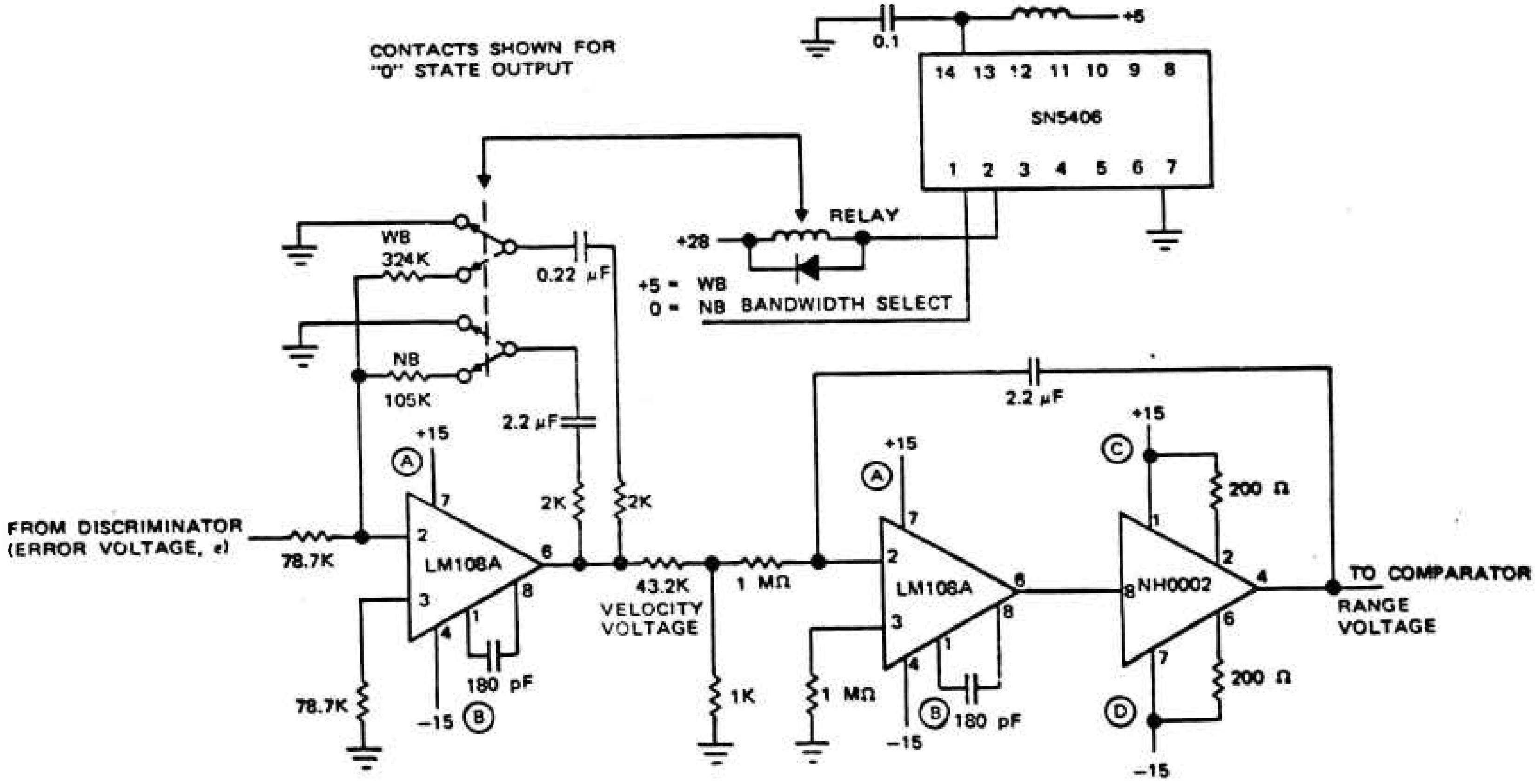


FIG. 18. Second-Order, Type 2 Range Track Integrator.

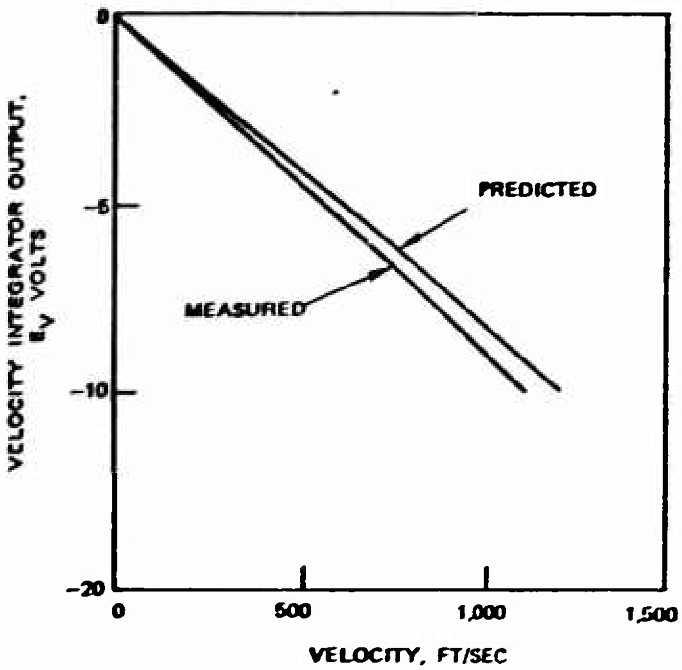
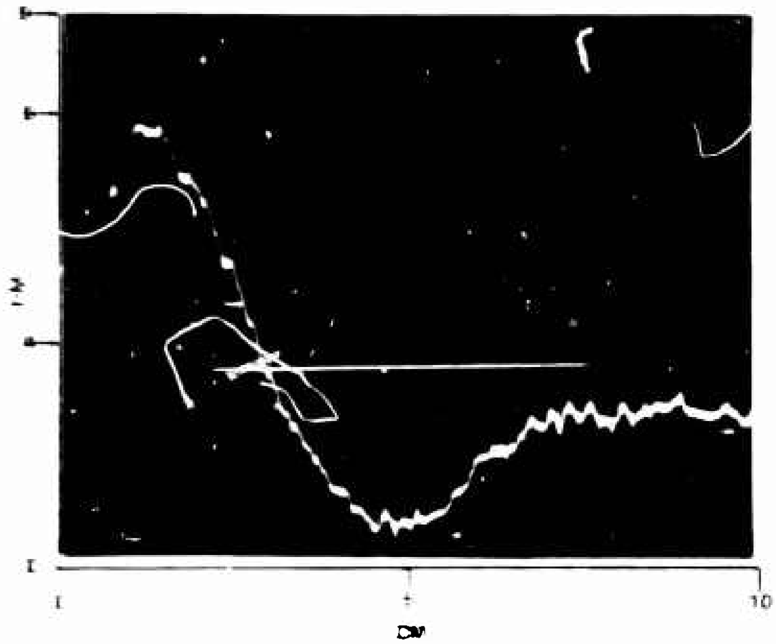
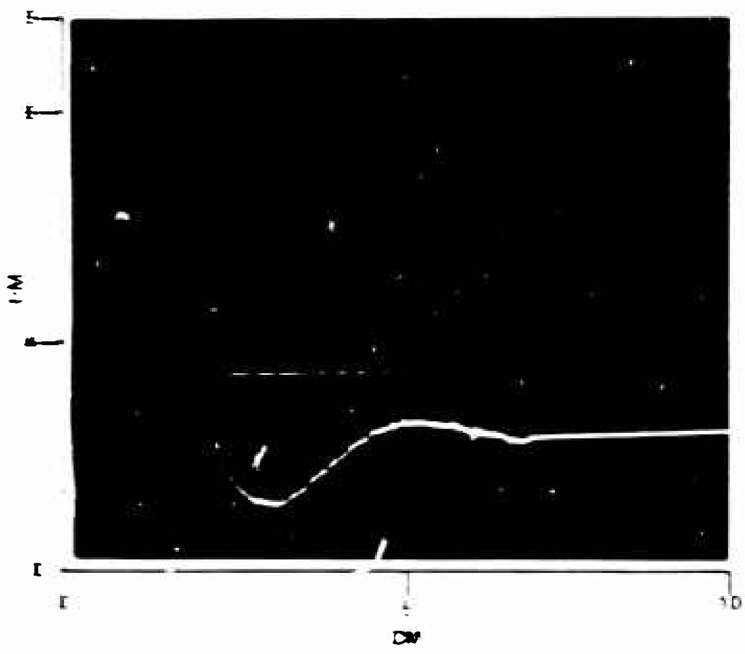


FIG. 19. Velocity Integrator Voltage Versus Velocity.

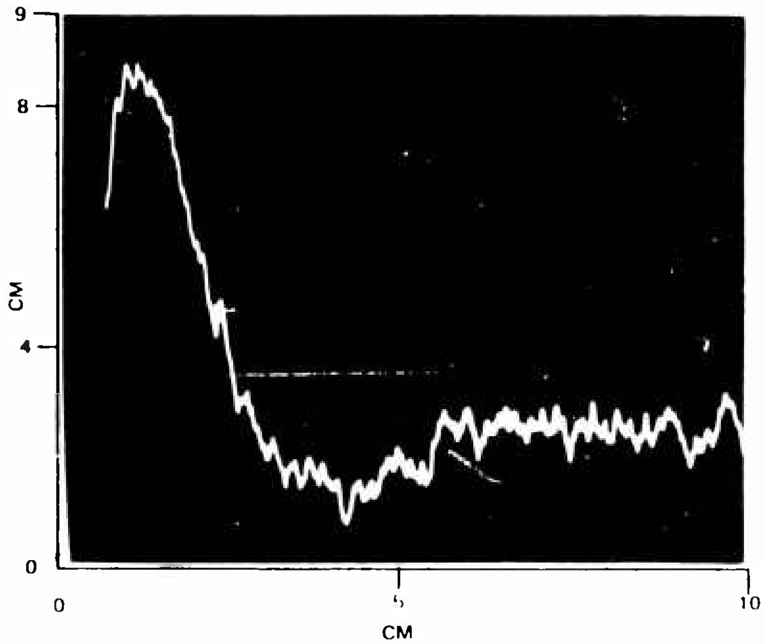


(a) Wide band; 300 ft/sec; 0.05 V/cm; 0.1 sec/cm; video input, [illegible] V.

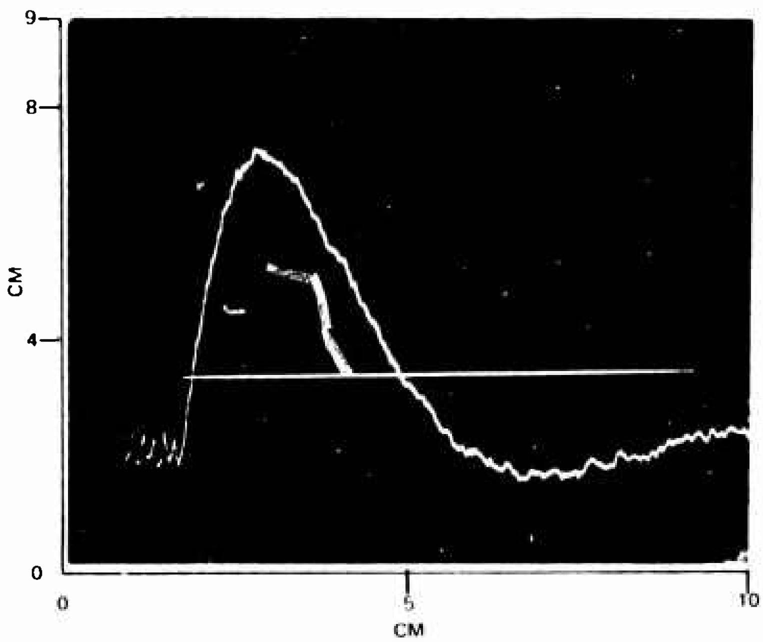


(b) Narrow band; 300 ft/sec; 0.2 sec/cm; video input, 1 V.

FIG. 21. Error Voltage (e) Versus Time for ΔVel = 300 foot/second, I = [illegible] milliwatt input.



(a) Wide band; 300 ft/sec; 0.05 V/cm; 0.1 sec/cm; video input, 1 V.



(b) Narrow band, 300 ft/sec, 0.2 V/cm, 0.5 sec/cm, video input, 1 V.

FIG. 21. Error Voltage (ε) Versus Time for ΔVel = 300 foot/second, D = 15 millivolt/foot.

CONCLUSIONS

This report has presented a simple approach to the design of second-order, type 2 range trackers. The validity of the design equations is demonstrated by close correlation between measured and predicted values of ω_n and δ for a design example.

Appendix A

RANGE TRACK LOOP/PHASE LOCKED LOOP COMPARISON

It was stated that the range track loop is essentially a phase locked loop. A brief discussion will now be given to clarify the similarities.

Figure 22 illustrates a basic phase locked loop. The loop input has a phase θ_i, which is compared with the output phase, f The difference in integrated, F(S), and the resulting voltage, V_2, drives the voltage controlled oscillator (VCO). The nature of feedback ensures that after loop closure

$$\theta_o = \theta_i \tag{61}$$

The error voltage, ε, may be given as

$$\varepsilon = K_d(\theta_i - \theta_o) \tag{62}$$

where

K_d = phase detector gain factor (volts/radian)

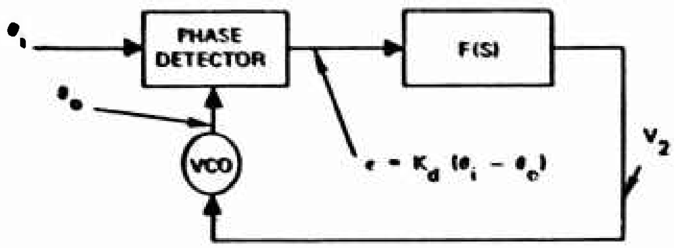


FIG. 22. Basic Phase Locked Loop.

Figure 23 illustrates the basic range track loop. The input to the discriminator is essentially a voltage that represents true range (see Fig. 6 and 7). This range-voltage conversion is accomplished via the early-late gates, voltage ramp, and comparator (see Fig. 5). The error voltage, ε, is

$$\varepsilon = D(R_i - R_o) \tag{63}$$

where

D = discriminator gain function (volt/foot)

Thus, the range tracker is nothing more than a phase locked loop with inputs of feet rather than radians.

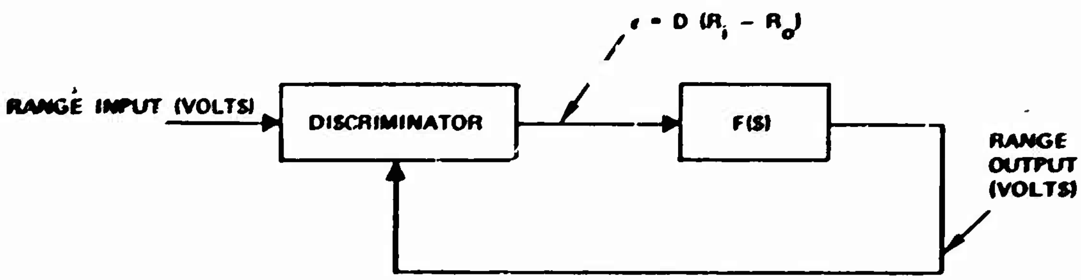


FIG. 23. Basic Range Track Loop.

Appendix B

RANGE TRACK LOOP SCHEMATICS

Figures 24–26 illustrate the range ramp generator, early-late gates, and range error discriminator and comparator. These circuits are connected as shown in Fig. 6.

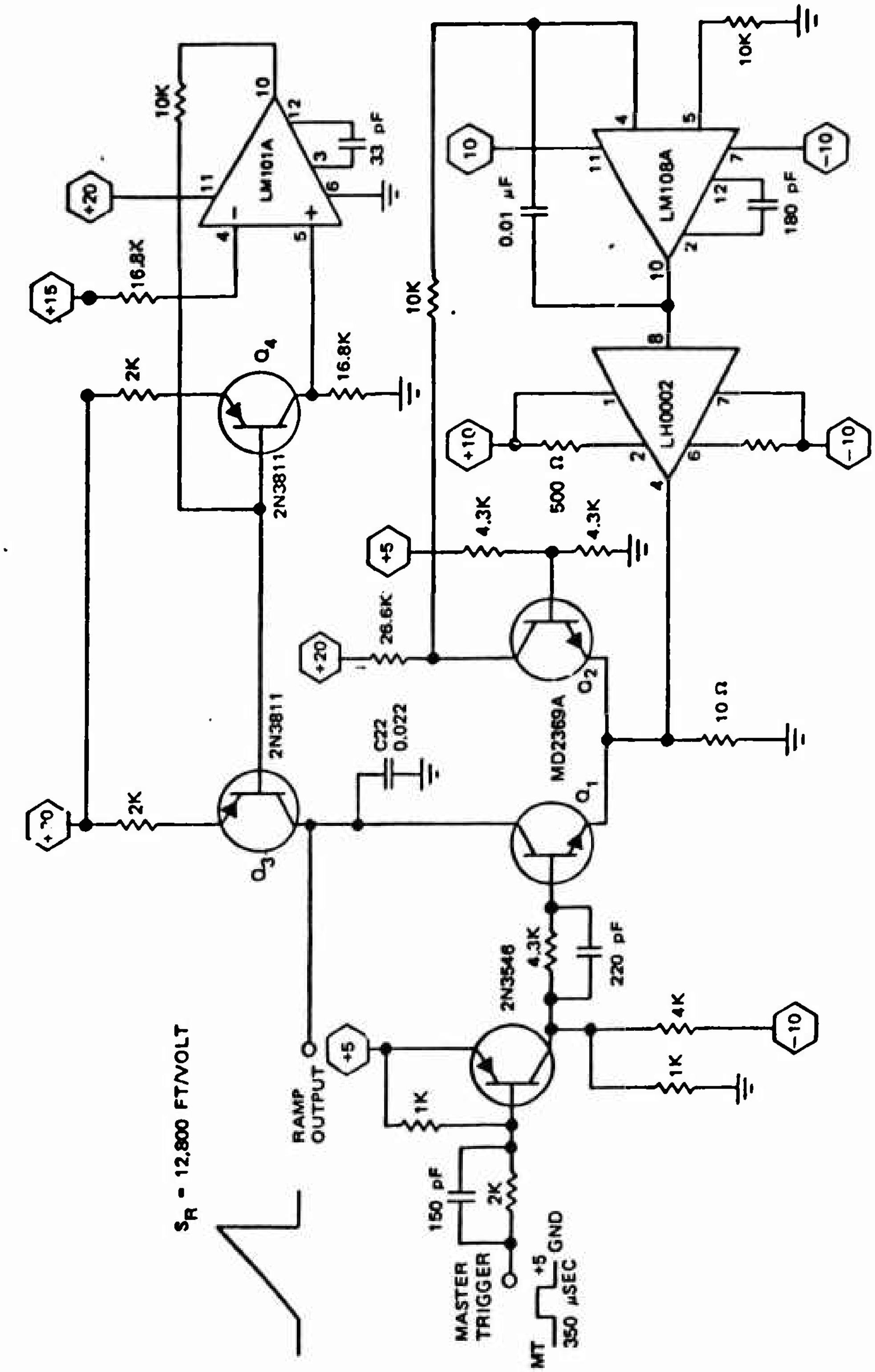


FIG. 24. Range Ramp Generator.

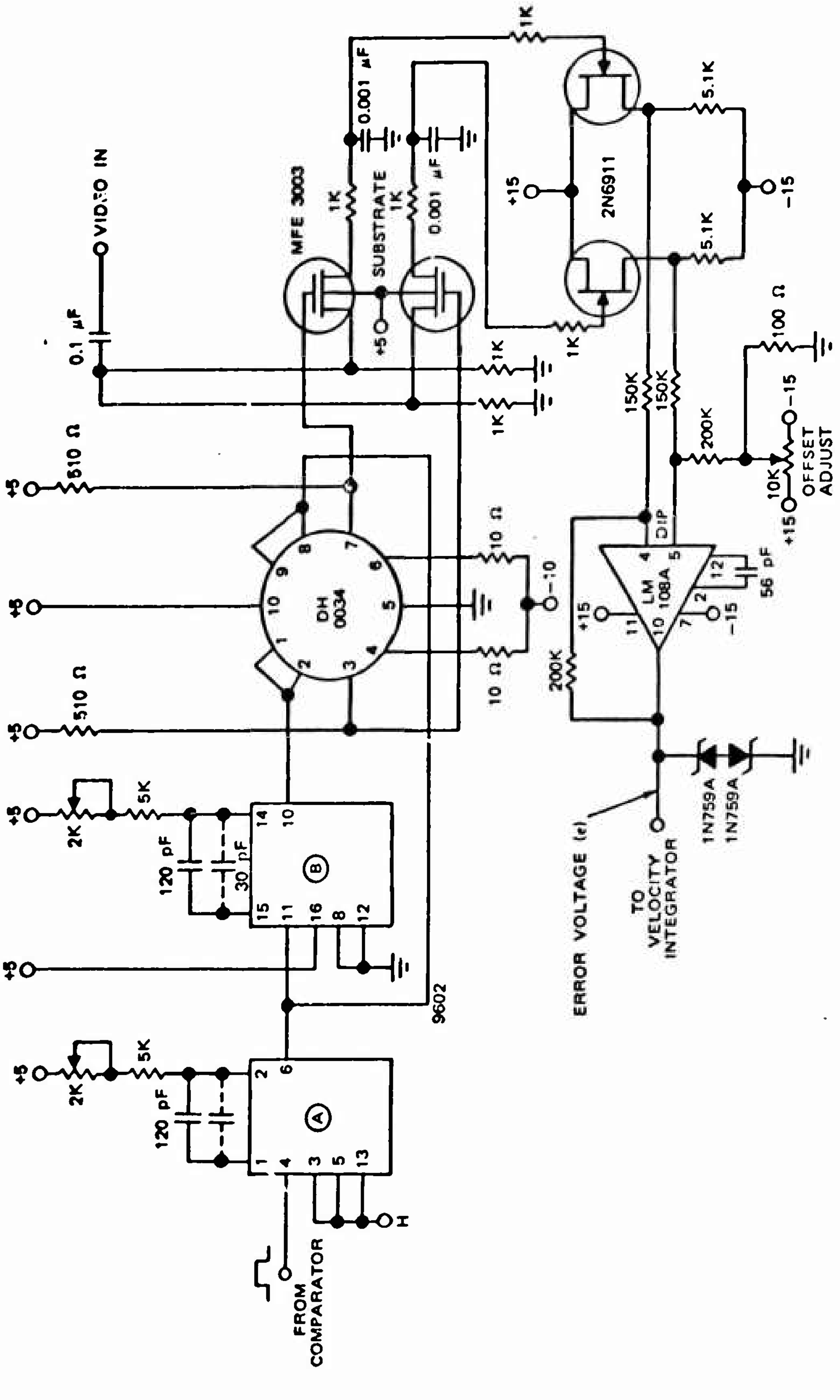


FIG. 25. MOS Early-Late Gates and Range Error Discriminator.

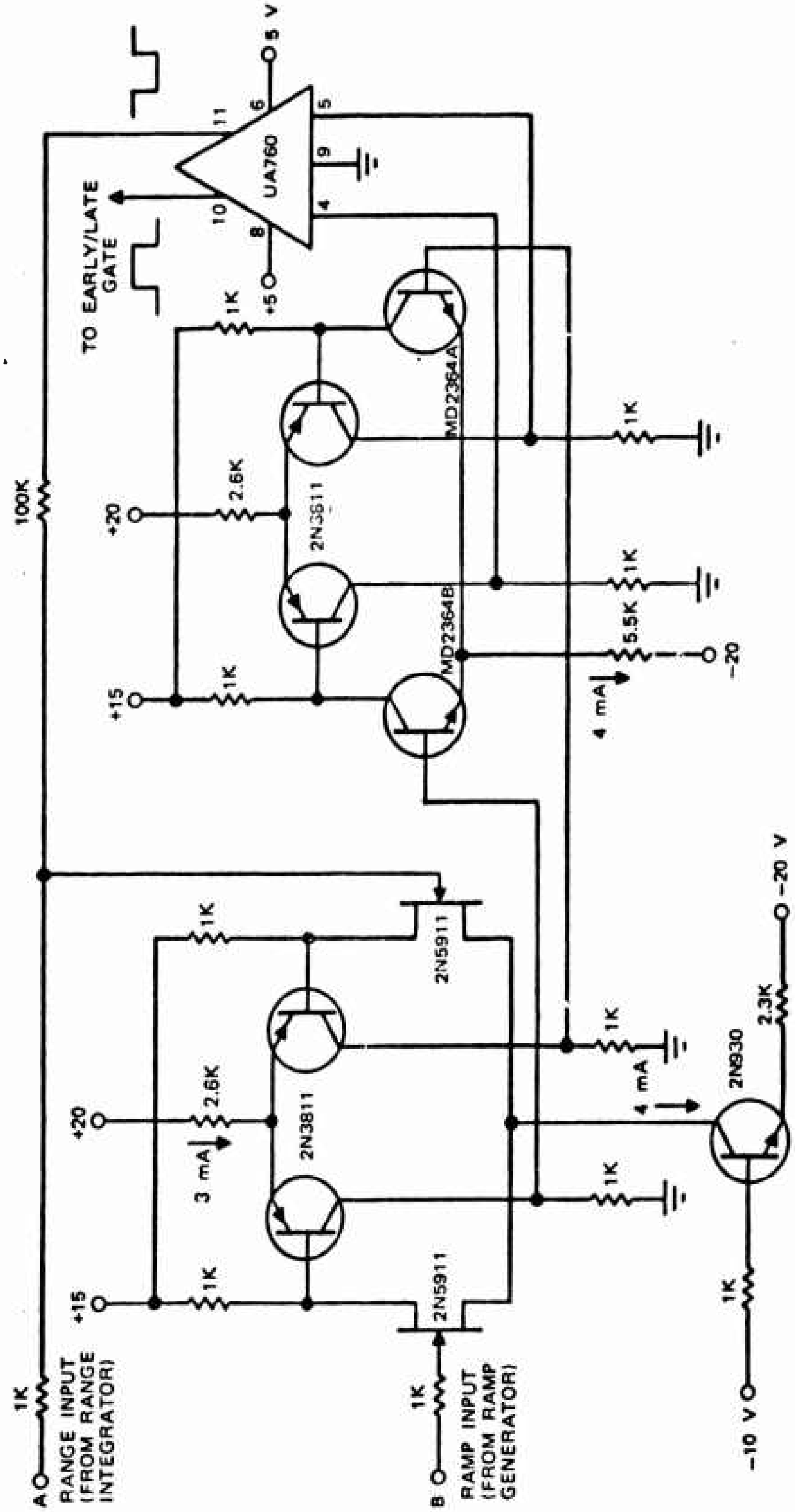


FIG. 26. Comparator.

NOMENCLATURE

A_1	$1/R_1C_1$ (Where R_1C_1 is the velocity integrator time constant)
A_2	$1/R_3C_2$ (Where R_3C_2 is the range integrator time constant)
A_3	R_2/R_1 (Velocity integrator gain)
Accel	Acceleration
a_1	$A_1A_2S_R$
a_2	$A_2A_3S_R$
BW/N	Noise bandwidth
C	Speed of light (0.984 ft/nsec)
D	Discriminator scale factor (volts/foot)
E_R	Range voltage (volts)
E_V/max	Maximum velocity integrator voltage for V_{max}
E_V'	$R_bE_V/max/R_b + R_a$
K	Constant $(2\delta/D)$
LG	Loop gain
R_A	Actual range to target
R_M	Measured range to target
S	Laplacian S
S_R	Scale factor (ft/volt)
S_V	Range voltage slope (volts/sec)
V_{max}	Maximum closing velocity
δ	Damping ratio
ω_n	Loop undamped natural frequency
ϵ	Range error (ΔR)
ΔR	Range error (ϵ)
ΔVel	Maximum velocity step

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